(8 pages) Reg. No. : .....

Code No. : 30741 E **Sub. Code : ESMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Third Semester

Mathematics

Skill Enhancement Course — COMPUTATIONAL MATHEMATICS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the Regula Falsi method, the new approximation x_{n+1} is computed based on ———
   (a) linear interpolation
   (b) quadratic interpolation
   (c) cubic interpolation
   (d) exponential interpolation

2. Choose the transcendental equation from the following ———
   (a) $x^3 - 1 = 0$ (b) $x^2 + x + 1 = 0$
   (c) $x = 1$ (d) $e^x - 1 = 0$

3. The order of convergence in Newton – Raphson method is ———
   (a) 3 (b) 2
   (c) 1 (d) 4

4. Horner's method is to find
   (a) Exact values of the roots of quadratic equation
   (b) Approximate values of the real roots of an equation
   (c) Approximate values of complex roots
   (d) The positive real roots of an equation

5. What is the system of simultaneous equation?
   (a) single equation with multiple variable
   (b) multiple equations with a single variable
   (c) multiple equations with multiple variables
   (d) an equation involving complex numbers

6. The Gauss – Jordan method reduces a original matrix into a ———
   (a) Identity matrix
   (b) Lower triangular matrix
   (c) Diagonal matrix
   (d) Upper triangular matrix

7. Which method is said to be direct method

(a) Gauss Seidal method

(b) Gauss Jacobi method

(c) Gauss Jordan method

(d) All the above

8. Gauss Seidal iteration converges only if the coefficient matrix is

(a) upper triangular

(b) lower triangular

(c) diagonally dominant

(d) banded matrix

9. In solving the Laplace equation $U_{xx} + U_{yy} = 0$, the standard five point formula is

(a) $U_{i,j} = \frac{1}{4}\left[U_{i+1,j+1} + U_{i+2,j-1} + U_{i-1,j-1} + U_{i-1,j+1}\right]$

(b) $U_{i,j} = \frac{1}{4}\left[U_{i-1,j} + U_{i+1,j} + U_{i,j-1} + U_{i,j+1}\right]$

(c) $U_{i,j} = \frac{1}{4}\left[U_{i,j+1} + U_{i,j-1} + U_{i-1,j-1} + U_{i-1,j+1}\right]$

(d) $U_{i,j} = \frac{1}{4}\left[U_{i+1,j+1} + U_{i+1,j-1} + U_{i-1,j+1} + U_{i-1,j-1}\right]$

10. The partial differential equation $\frac{\partial^2 U}{\partial x^2} + 2\frac{\partial^2 U}{\partial x \partial y} + 3\frac{\partial^2 U}{\partial^2 y} = 0$ is

(a) Hyperbolic

(b) Elliptic

(c) Parabolic

(d) Rectangular hyperbola

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Use the method of iteration to solve the equation $3x - \log_{10} x = 6$.

Or

(b) Can we apply iteration method to find the root of the equation $2x = \cos x + 3$ in $\left[0, \frac{\pi}{2}\right]$?

12. (a) Explain the method of Bisection.

Or

(b) Find the real root of $x^3 - 3x + 1 = 0$ lying between 1 and 2 upto three places of decimals by Newton Raphson method.

13. (a) Solve the following system of equations using Gauss elimination method : $x + y + z = 9$; $2x - 3y + 4z = 13$; $3x + 4y + 5z = 40$.

Or

(b) Solve the following system of equations by Gauss Jordan method $5x - 2y + 3z = 18$, $x + 7y - 3z = -22$, $2x - y + 6z = 22$.

14. (a) Solve $2x + y = 3$; $2x + 3y = 5$ by Gauss Seidel iteration method.

Or

(b) Solve the following equations using relaxation method

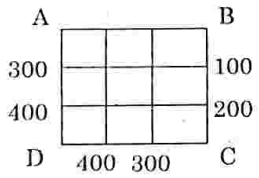
$5x - y - z = 3$; $-x + 10y - 2z = 7$;

$-x - y + 10z = 8$.

15. (a) Classify the equation $u_{xx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = \sin xy$.

Or

(b) Solve the equation $U_{xx} + U_{yy} = 0$ for the following square mesh with boundary values as shown below using Liebmann method.



PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Find the real root lying between 1 and 2 of the equation $x^3 - 3x + 1 = 0$ upto 3 places of decimal by using Regula Falsi method.

Or

(b) Find the real root of the equation $\cos x = 3x - 1$ correct to four places of decimals using successive approximation method.

17. (a) Find the real root of $xe^x - 2 = 0$ correct to three places of decimals using Newton Raphson method.

Or

(b) Find the negative root of $x^3 - x^2 + 12x + 24 = 0$ correct to two places of decimals by Horner's method.

18. (a) Find the inverse of the matrix by Gauss elimination $A = \begin{pmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$.

Or

(b) Solve the following system of equations by Gauss Jordan method :

$x + y + z = 9$; $2x - 3y + 4z = 13$;

$3x + 4y + 5z = 40$.

19. (a) Solve the following equations using Jacobi's iteration method. $28x + 4y - z = 32$;

$x + 3y + 10z = 24$; $2x + 17y + 4z = 35$.

Or

(b) Solve the following system of equations using Gauss Seidal iteration method.

$6x + 15y + 2z = 72$; $x + y + 54z = 110$;

$27x + 6y - z = 85$.

20. (a) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 8x^2y^2$ in the square mesh given $u = 0$ on the four boundaries dividing the square into 16 subsquares of length 1 unit.

Or

(b) By iteration method solve the elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ over the square region of side 4 satisfying the boundary conditions.

(i) $u(0, y) = 0$ for $0 \le y \le 4$

(ii) $u(4, y) = 12 + y$ for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$ for $0 \le x \le 4$

(iv) $u(x, 4) = x^2$ for $0 \le x \le 4$.

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